Math 115

Winter 2017

Lecture 13

Use Special products

$(A+B)^2 = A^2+2AB+B^2$, $(A-B)^2 = A^2-2AB+B^2$,

and $(A+B)(A-B) = A^2-B^2$ to find

① $(2x^3+5)^2$

$= (2x^3)^2 + 2(2x^3)(5) + (5)^2$

$= \boxed{4x^6 + 20x^3 + 25}$

② $(3x^2-4y^3)^2$

$= (3x^2)^2 - 2(3x^2)(4y^3) + (4y^3)^2$

$= \boxed{9x^4 - 24x^2y^3 + 16y^6}$

③ $(x^4+x^2)(x^4-x^2)$

$= (x^4)^2 - (x^2)^2$

$= \boxed{x^8 - x^4}$

Divide:

① $\dfrac{72x^9y^5 - 64x^4y^6}{-8x^4y^5}$

$= \dfrac{72x^9y^5}{-8x^4y^5} - \dfrac{64x^4y^6}{-8x^4y^5}$

$= \boxed{-9x^5 + 8y}$

② $\dfrac{3x^3 - 4x^2 + 8}{x-2}$

$3x^2 + 2x + 4$

$x-2 \overline{)\, 3x^3 - 4x^2 + 0x + 8}$

$-(3x^3 - 6x^2)$

$2x^2 + 0x + 8$

$-(2x^2 - 4x)$

$4x + 8$

$-(4x - 8)$

16

$x \cdot \boxed{3x^2} = 3x^3$

$x \cdot \boxed{2x} = 2x^2$

$x \cdot \boxed{4} = 4x$

$3x^2 + 2x + 4 + \dfrac{16}{x-2}$

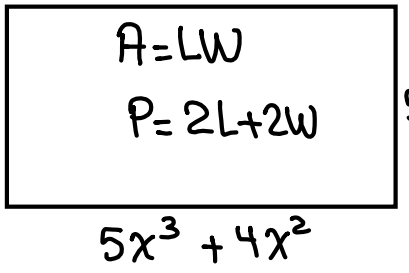
Simplify

① $(9.5 \times 10^{17}) \cdot (5.6 \times 10^{12})$

$= 53.2 \times 10^{29} = 5.32 \times 10^{1} \times 10^{29} = \boxed{5.32 \times 10^{30}}$

② $\dfrac{4.2 \times 10^{-8}}{1.68 \times 10^{15}} = 2.5 \times 10^{-8-15} = \boxed{2.5 \times 10^{-23}}$

Find A & P:



$A = LW$

$= (5x^3+4x^2)(5x^3-4x^2)$

conjugates

$= (5x^3)^2 - (4x^2)^2$

$A = 25x^6 - 16x^4$

$P = 2L + 2W$

$= 2(5x^3+4x^2) + 2(5x^3-4x^2)$

$= 10x^3 + 8x^2 + 10x^3 - 8x^2$

$P = 20x^3$

Divide:

$$\frac{2x^3 - x^2 - 13x - 5}{2x + 5}$$

$2x \boxed{x^2} = 2x^3$

$2x \boxed{-3x} = -6x^2$

$2x \boxed{1} = 2x$

$$\begin{array}{r} x^2 - 3x + 1 \\ 2x+5 \overline{)\,2x^3 - x^2 - 13x - 5} \\ -(2x^3 + 5x^2) \\ \hline -6x^2 - 13x - 5 \\ -(-6x^2 - 15x) \\ \hline 2x - 5 \\ -(2x + 5) \\ \hline -10 \end{array}$$

$$x^2 - 3x + 1 + \frac{-10}{2x+5}$$

Multiply

$(x+3)(x-3)(x^2+9)$

$= (x^2-9)(x^2+9) = (x^2)^2 - 9^2 = \boxed{x^4-81}$

$(2x-1)(2x+1)(4x^2+1)$

$= (4x^2-1)(4x^2+1) = (4x^2)^2 - (1)^2$

$= \boxed{16x^4-1}$

$A = 27x^3 - 8$

$3x-2$

Find the length

$A = LW$

$27x^3 - 8 = L(3x-2)$

$L = \dfrac{27x^3-8}{3x-2}$

$$\begin{array}{r} 9x^2+6x+4 \\ 3x-2 \overline{)\,27x^3+0x^2+0x-8} \\ -(27x^3-18x^2) \\ \hline 18x^2+0x-8 \\ -(18x^2-12x) \\ \hline 12x-8 \\ -(12x-8) \\ \hline 0 \end{array}$$

$3x \boxed{9x^2} = 27x^3$

$3x \boxed{6x} = 18x^2$

$3x \boxed{4} = 12x$

The length is $9x^2+6x+4$

We need 40 Liters of 22% alcohol Soln.
We have unlimited supply of 15% & 25% alcohol Solns. Use system of linear eqns to find how many liters of each?

15%	+	25%	=	22%
x		y		40

$$\begin{cases} x + y = 40 \\ \frac{15}{100}x + \frac{25}{100}y = \frac{22}{100}\cdot 40 \end{cases} \quad \times 100$$

$$\begin{cases} x + y = 40 \\ 15x + 25y = 22 \cdot 40 \end{cases} \quad \div 5$$

$$\begin{cases} x + y = 40 \\ 3x + 5y = 176 \end{cases} \quad -3$$

$$2y = 56$$

$y = 28$

28L of 25%
&
12L of 15%

$195 in total simple interest in 1 yr.
Two accounts, one pays 5%, another one pays 8%.
The money in 8% account was $100 less than 3 times the money in 5% account. Use system of linear eqns to find how much per account.

Total interest = 195

5%	+	8%	
x		y	

$$\begin{cases} \frac{5}{100}x + \frac{8}{100}y = 195 \\ y = 3x - 100 \end{cases} \quad \times 100$$

$$\begin{cases} 5x + 8y = 19500 \\ y = 3x - 100 \end{cases}$$

$5x + 8(3x - 100) = 19500$

$5x + 24x - 800 = 19500$

$29x = 19500 + 800$

$29x = 20300$

$x = \frac{20300}{29}$

$\rightarrow x = 700$

$700 @ 5\%$

&

$2000 @ 8\%$

Factor out the GCF:

1) $3x + 15$

$= 3(x + 5)$

2) $5x^2 - 10x$

$= 5x(x - 2)$

3) $20x^3 - 30x^2 - 10x$

$= 10x(2x^2 - 3x - 1)$

4) $7x(2x-3) - 5(2x-3)$

$= (2x-3)(7x - 5)$

Factor by grouping

① $x^3 + 2x^2 + 4x + 8$

$= x^2(x+2) + 4(x+2) = \boxed{(x+2)(x^2+4)}$

② $5x^3 - 3x^2 - 25x + 15$

$= x^2(5x-3) - 5(5x-3)$

$= \boxed{(5x-3)(x^2-5)}$

Factor completely

① $x^2 + 7x + 12 = x^2 + 3x + 4x + 12$

$P = 12$, $S = 7$

$1, 12$

$2, 6$

$3, 4$

$= x(x+3) + 4(x+3)$

$= \boxed{(x+3)(x+4)}$

② $x^2 + x - 12 = x^2 - 3x + 4x - 12$

$P = -12$, $S = 1$

$-1, 12$

$-2, 6$

$-3, 4$

$= x(x-3) + 4(x-3)$

$= \boxed{(x-3)(x+4)}$

③ $2x^2 + 7x + 5 = 2x^2 + 2x + 5x + 5$

$P = 10$, $S = 7$

10: $1, 10$; $2, 5$

$= 2x(x+1) + 5(x+1)$

$= (x+1)(2x+5)$

④ $2x^2 - 3x - 5 = 2x^2 - 5x + 2x - 5$

$P = -10$, $S = -3$

-10: $1, -10$; $2, -5$

$= x(2x-5) + 1(2x-5)$

$= (2x-5)(x+1)$

⑤ $x^2 - 13x + 36 = x^2 - 4x - 9x + 36$

$P = 36$, $S = -13$

36: $-1, -36$; $-2, -18$; $-3, -12$; $-4, -9$; $-6, -6$

$= x(x-4) - 9(x-4)$

$= (x-4)(x-9)$

⑥ $x^2 - 12x + 36 = x^2 - 6x - 6x + 36$

$P = 36$, $S = -12$

36

$= x(x-6) - 6(x-6)$

$= (x-6)(x-6)$

$= (x-6)^2$

Special Factoring

$A^2 + B^2 \rightarrow$ Prime, $A^2 - B^2 = (A+B)(A-B)$

① $x^2 + 49$

$= x^2 + 7^2 \rightarrow$ Prime

② $x^2 - 100$

$= x^2 - 10^2$

$= (x+10)(x-10)$

③ $25x^2 - 64$

$= (5x)^2 - (8)^2$

$= (5x+8)(5x-8)$

④ $36x^2 - 49y^2$

$= (6x)^2 - (7y)^2$

$= (6x+7y)(6x-7y)$

Factor completely

① $16x^2(5x-3) - 25(5x-3)$

$= (5x-3)(16x^2-25) = (5x-3)(4x+5)(4x-5)$

$(4x)^2 - (5)^2$

② $x^2(7x+1) - 6x(7x+1) - 16(7x+1)$

$= (7x+1)(x^2 - 6x - 16)$

1, 16
2, 8
4, 4

$= (7x+1)(x-8)(x+2)$

Use $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Factor

① $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$

② $8x^3 - 125 = (2x)^3 - 5^3$

$= (2x-5)(4x^2 + 10x + 25)$

Factor

$27x^3(3x-10) - 1000(3x-10)$

$= (3x-10)(27x^3 - 1000)$

$(3x)^3 - (10)^3$

$= (3x-10)(3x-10)(9x^2 + 30x + 100)$

$= (3x-10)^2(9x^2 + 30x + 100)$

$$64x^3(64x^2-25)+125(64x^2-25)$$

$$=(64x^2-25)(64x^3+125)$$

$(8x)^2-5^2$ $\qquad$ $(4x)^3+5^3$

$$=\boxed{(8x-5)(8x+5)(4x+5)(16x^2-20x+25)}$$

Since $0\cdot\#=0$

$0(2x-3)=0$, $(3x+5)\cdot 0=0$

Solve $(3x+5)(2x-3)=0$

$3x+5=0$ or $2x-3=0$

$3x=-5$ $\qquad$ $2x=3$

$\boxed{x=\frac{-5}{3}}$ $\qquad$ $\boxed{x=\frac{3}{2}}$

Final Ans

$\left\{\frac{-5}{3},\frac{3}{2}\right\}$

Zero-Product Rule or Zero-Factor Thrm.

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (Maybe both)

Solve

$$(x-7)(x+10)=0$$

$x-7=0$ OR $x+10=0$

$\boxed{x=7}$ $\boxed{x=-10}$ $\{-10, 7\}$

Solve Polynomial eqn:

1) RHS = 0

2) LHS must be factored

3) Use Z.F.T., and solve each factor.

Solve

$x^2-10=3x$

$x^2-10-3x=0$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

by Z.F.T.

$x-5=0$ or $x+2=0$

$\boxed{x=5}$ or $\boxed{x=-2}$ $\Leftarrow$ $\{-2, 5\}$

Solve

$(x+4)(x-2) = -5$

1) Foil & Simplify

$x^2 - 2x + 4x - 8 + 5 = 0$

$x^2 + 2x - 3 = 0$

2) Factor LHS

$(x+3)(x-1) = 0$

3) Use Z.F.T. $x+3=0$ or $x-1=0$

$\boxed{x=-3}$ $\boxed{x=1}$

$\{-3, 1\}$

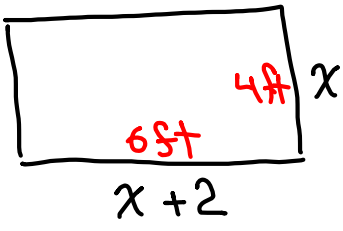
The length of a rectangle is 2 ft longer than its width. Area is 24 ft^2.

Find its dimensions.

$A = 24$

$LW = 24$

4ft by 6ft



$x(x+2) = 24$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

by Z.F.T.

$x+6=0$ or $x-4=0$

~~$x=-6$~~ $x=4$

Solve

$3x^2 - 4 = x$

① RHS = 0 $\quad 3x^2 - 4 - x = 0$

② LHS must be factored $\Rightarrow 3x^2 - x - 4 = 0$

$\Downarrow$

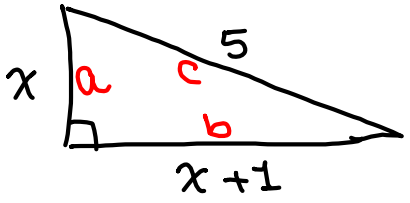
$(3x - 4)(x + 1) = 0$

by Z.F.T.

$3x - 4 = 0 \quad x + 1 = 0$

$x = 4/3 \quad x = -1$

Find x:



① Right Triangle

② use Pythagorean Thm

$a^2 + b^2 = c^2$

$(A+B)^2$

$x^2 + (x+1)^2 = 5^2$

$x^2 + x^2 + 2x + 1 = 25$

$2x^2 + 2x + 1 - 25 = 0$

$2x^2 + 2x - 24 = 0$

Divide by 2

→ by Z.F.T.

~~$x = -4$~~, $\{3\}$

$\boxed{x = 3}$

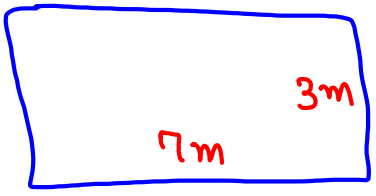
$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

Area of a rectangle is $21\ m^2$.

length is 1 m longer than twice its width. Find its dimensions

3m by 7m



$w = x$

$L = 2x + 1$

$A = 21$

$LW = 21$

$(2x+1) \cdot x = 21$

$2x^2 + x - 21 = 0$

$(2x + 7)(x - 3) = 0$

by Z.F.T.

$2x + 7 = 0$ or $x - 3 = 0$

$x = -\frac{7}{2}$ / $x = 3$